NOTA BENE: I have no background in finance, and I have never worked in the accounting, finance, or real estate industries. While I believe the mathematical manipulations themselves to be correct, the equations presented here are based on assumptions which may not be consistent with actual practice. If anyone chooses to make use of this work, industry analysts should be consulted to confirm these results. Corrections or suggestions are welcome.

I've defined the following variables to derive the original amortization equation:

P	The principal borrowed
N	The number of payments
i	The fractional (periodic) interest rate
P_j	The principal portion of payment j
B	A final balloon payment
x	The regular payment

The result of the derivation is this equation for x, the periodic payment [Eq. (6) in the derivation document]:

$$x = i\left[\frac{P(1+i)^N}{(1+i)^N - 1} + \frac{B}{(1+i)-(1+i)^{N+1}}\right]. \qquad (6)$$

The periodic interest rate i is calculated from the annual interest rate (APR) and the number of payments made annually. E.g., for monthly payments, $i = APR/12$, which is equivalent to the traditional calculation for a 360-day basis.

If there is a moratorium period before the amortization payment schedule takes effect, then additional interest may accrue, either simple or compounded. Unless other payment options are contracted, when the moratorium period ends, the amortization may simply treat the accrued interest as additional principal. So if m represents the number of (non-)payment cycles in the moratorium period, the adjusted principal amount (P') would be calculated as

$$P' = P \cdot (1+i)^m \qquad (7a)$$

for compounded interest, or calculated as

$$P' = P \cdot (1+mi) \qquad (7b)$$

for simple interest. These equations apply for any non-negative m: if $m = 0$, then $P' = P$. The amortization equation could then always use the adjusted principal (P') instead:

$$x = i\left[\frac{P'(1+i)^N}{(1+i)^N - 1} + \frac{B}{(1+i)-(1+i)^{N+1}}\right]. \qquad (8)$$

Note that N represents the number of payments in the amortization schedule only; that is, the number of payment cycles in the moratorium period are not included.

The balloon payment B is expected to occur one payment cycle after the end of the amortization, and is useful in the calculation if one knows ahead of time how much one wishes to payout when amortization ends. If the balloon is zero, then the financed amount is entirely paid off after N payments.

If the repayment schedule is to be terminated early, i.e., before N payments have been made, we must first determine the principal balance. After r payments, we can see that the principal balance R will be

$$R = P' - P'_1 - P'_2 - \cdots - P'_r = P' - \sum_{j=1}^{r} P'_j. \qquad (9)$$

Eq. (2) from the amortization derivation shows that $P_j = (x - iP)(1+i)^{j-1}$, so the principal balance may also be expressed as

$$R = P' - \sum_{j=1}^{r}(x - iP')(1+i)^{j-1}. \qquad (10)$$

We can rewrite the limits of summation,

$$R = P' - (x - iP')\sum_{j=0}^{r-1}(1+i)^j, \qquad (11)$$

and then we may employ the same series-elimination transformation described in the original derivation document to get

$$R = P' - (x - iP')\frac{(1+i)^r - 1}{i}. \qquad (12)$$

Assuming that our early payoff happens one payment cycle after the last regular payment, the final payment including interest would be the amount

$$R \cdot (1+i). \qquad (13)$$

Equation (12) is quite useful in that it holds for any payment amount x. If we allow $x < iP'$, then the principal balance will grow over time, which makes sense since the payment doesn't cover the interest due on the original principal balance. If $x = iP'$, then we are making interest-only payments, so the principal balance does not change. Only with $x > iP'$ will we reduce the principal balance over time. Eq. (8) allows us to specify the payment amount in terms of the time span, so that if we've set x accordingly, the principal balance will reach zero when $r = N$ (provided $B = 0$; otherwise the principal balance will be B when $r = N$).

Note that all of these equations implicitly define exact quantities, potentially in fractions of a cent. In the real world, we normally pay in whole pennies, so payments are usually rounded to the nearest cent. Because of the compounding nature of amortization interest calculations, it is not always easy to predict how much effect the rounding will have by the end of the amortization period. Someday I'll do my own error analysis, which would allow me to define R (for example) to account for rounding. □